

A 30 to 40 Billion Year Old Universe

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Summary

The Friedmann (1922) equations are a solution for the expansion of the universe from Einstein's general relativity equations as applied to cosmology. The temperature history of the universe is reviewed to show how it is believed that the observable universe has expanded by 3000K/2.7K ~ 1100 since it became transparent to radiation. However, this would have existed ~13.8 billion years ago, and the universe should have continued to expand since then, perhaps to a "radius" of ~14.4 Gpc. In order to back out reasonable values of Hubble's constant, to compare with verified measured values out to a distance of ~3 Gpc from the Sun, the age of the universe must be substantially older than current estimates.

The Friedmann Equations

The two Friedmann equations derived from Einstein's (1917) cosmological equations are [Friedmann (1922), Swihart (1968)]:

$$\dot{R}^2 + 2R\ddot{R} + kc^2 = -8\pi GpR^2/c^2 + \Lambda R^2 \quad (1)$$

$$\dot{R}^2 = \frac{8\pi G\rho R^2}{3} - kc^2 + \frac{\Lambda R^2}{3} \quad (2)$$

where a single dot over the R is the first derivative, dR/dt; the double dot is the second derivative; R is the radius of curvature of space; t time; k either -1, 0, +1 depending on if the universe has negative, flat (Euclidean), or positive curvature; c the speed of light; Λ Einstein's fabricated cosmological constant; G the universal gravitational constant; p pressure; and ρ average density (including both observed matter as well as dark matter).

The 2nd equation is all that is needed to predict the universe's expansion versus time, as well as its age. Note that the only inputs required are the average density at time t, k, and the cosmological "constant"; the other values are physics constants.

The Size of the Universe when it Became Transparent

Equation (2) can be simplified during the time that radiation dominated over matter, since the cosmological term is negligible at early times, following similar steps as Weinberg (1972):

$$\dot{R}^2 = 8\pi G\rho_r R^2/3 \quad (3)$$

where ρ_r is the radiation density. Also, in an adiabatic expansion, the temperature T_r of the fireball is inversely proportional to R^2 for non-relativistic speeds, but to R for relativistic speeds

[Weinberg (1972); Rowan-Robinson (2004)], so in terms of initial conditions,

$$R = T_i R_i / T_r \quad (4)$$

At the extreme temperatures of the fireball when radiation dominated over matter, the radiation pressure, $\rho_r c^2/3$, causes the relativistic expansion. If ρ_r was zero, there would be no expansion of space, refuting recent beliefs. Differentiating and substituting into equation (3) results in:

$$\dot{R} = -T_i R_i \dot{T}_r / T_r^2 \quad (5)$$

$$\dot{T}_r^2 = \frac{8\pi}{3} G \rho_r T_r^2 \quad (6)$$

The radiation density ρ_r , following Weinberg (1972) and Berry (1976), is

$$\rho_r = \alpha T_r^4 / c^2 \quad (7)$$

where the black body constant, $\alpha = 4\sigma/c$, and σ is the Stefan-Boltzmann constant. Substituting these terms into equation (6) and taking the square root,

$$\dot{T}_r = \frac{dT_r}{dt} = \sqrt{\frac{8\pi G \alpha}{3c^2}} T_r^3 \quad (8)$$

Separating variables leads to the following differential equation,

$$\frac{dT_r}{T_r^3} = \sqrt{\frac{8\pi G \alpha}{3c^2}} dt \quad (9)$$

The solution for the temperature of the fireball versus time (t) then becomes:

$$T_r = \sqrt{\sqrt{\frac{3c^2}{32\pi G \alpha}} / \sqrt{t}} \quad (10)$$

or

$$T_r = \sqrt{2.31 \times 10^{20} \text{ seconds} - K^2} / \sqrt{t}$$

Weinberg goes into much more detail involving neutrinos, antineutrinos, electron-positron annihilation, relativistic effects, etc, deriving ~30% lower values for $T_r > 5 \times 10^9$ K. Temperature drops to $\sim 10^{10}$ K after one second when nucleosynthesis of hydrogen and helium can begin. After a few minutes, the temperature will drop to 10^9 K when nucleosynthesis will start to end, leaving nuclei of hydrogen, helium, and a little lithium. There is not enough time available to create heavier elements as exists in the center of stars. And, there is not enough tritium or lithium to continue nucleosynthesis at lower temperatures typical of thermonuclear weapons. Note that the energy release from nuclear fusion reactions or electromagnetic effects from plasmas, that should affect the expansion, are not included in this simplified analysis.

The fireball temperature drops to ~ 4000 K at 1.2×10^{13} seconds (380,000 yrs). This is the approximate temperature when the ionized plasma is no longer ionized and becomes transparent to radiation, whereas Rowan-Robinson (2004) and others use 3000 K. This is analogous to the ionized plasma that surrounded the Apollo spacecraft during reentry, which prevented communication via radio waves until the spacecraft fireball subsided. However, expansion speeds are still over an order of magnitude larger than the speed of light long after atoms are formed, and rapidly decreasing, violating special relativity but not general relativity.

Since the 2.7 K Cosmic Microwave Background (CMB) is believed to be redshifted from this fireball, the radius of the universe should have expanded by $3000 \text{ K} / 2.7 \text{ K} \sim 1100$ at the present day (see equation 4). If the current radius of the universe is 4.2 Gpc, then the radius of the CMB fireball would be 3.8 Mpc. If the current radius is 14.4 Gpc, then the CMB fireball would have a radius of 13 Mpc (or 26 Mpc diameter). For comparison, this latter size is larger than the Virgo cluster of galaxies and far larger than our local group of galaxies.

Similarly, by taking the ratio of $10^9 \text{ K} / 3000 \text{ K}$, we can estimate the universe's radius when nucleosynthesis completed as $4.2 \text{ Gpc} / 3.3 \times 10^5 = 13 \text{ pc}$ at a temperature of 10^9 K . For comparison, there are at least 100 stars within this radius from our Sun, including Arcturus, Vega, and Sirius.

It should be noted that the surface of the early, extremely hot fireball was capable of radiating to empty space based on its blackbody temperature vs time while it was optically opaque, and the cooler fireball ($\sim 3000 \text{ K}$) radiated from its entire volume when it became transparent (optically thin). The total power emitted from an optically opaque 13 pc (26 pc diameter) fireball at 10^9 K will exceed the power from a 13 Mpc (26 Mpc diameter) fireball at 4000 or 3000 K by over 10 orders of magnitude. However, we cannot see this radiation from a unique direction today because it would advance ahead of expanding matter.

The early expansion does not drop below light speed until after several hundred million years, violating special relativity long after hydrogen and helium are formed. General relativity, however, allows expansion speeds exceeding the speed of light – taking the square root of equation (2) calculates this speed vs time.

The Expansion of the Universe after it Became Transparent

The universe may be much older than 13.8 billion years, as expanding matter would have to average light speed to expand to 13.8 billion light years (4.2 Gpc) in this time. And, if the universe's radius is 14.4 Gpc as it has continued to expand, it would have to exceed three times the speed of light to expand this far in 13.8 billion years. In general relativity, this is considered acceptable. Let's explore several approaches to calculate the expansion after the time of last scattering ("recombination").

Expansion using Hubble's Law

If the universe is homogeneous, can Hubble's Law be used in reverse? If it were possible for an observer to be located at the Big Bang location, galaxies would be seen receding at the same relative rate as we see from our galaxy. In this case, the expansion is estimated from Hubble's Law, which states that velocity (from redshift measurements) is proportional to distance R [Hubble (1929)], assuming we can apply it in either a reverse or forward direction:

$$v = dR/dt = H_0 R \quad (11)$$

After separating variables, a simple differential equation is obtained:

$$dR/R = H_0 dt$$

H_0 is approximately a constant, neglecting the slight increase in acceleration measured by Reiss et. al. (1998) and Perlmutter et.al. (1999). Therefore, integration yields the time since the universe became transparent [Swihart (1968)]:

$$t = \frac{\ln(R/R_{\text{transparent}})}{H_0} \quad (12)$$

In order for the universe to expand by a ratio of $R/R_{\text{transparent}} = 1100$ since the universe became transparent, for $H_0 = 73 \text{ km/s/Mpc} = 2.36 \times 10^{-18}/\text{s}$, requires an elapsed time of 2.95×10^{18} seconds, or 94 billion years. The initial 380,000 years when the universe was opaque is insignificant in this estimate. An $H_0 = 67 \text{ km/s/Mpc}$, as estimated by analyzing the structure in the CMB and causing the so-called Hubble Tension [Schilling (2019)], would give an age of the universe over 100 billion years. Using a range of temperatures from 2000 to 5000 K to estimate when the universe became transparent will not change these overall conclusions.

Equation (12) can also be written to calculate radius as a function of time [Weinberg (1972)]:

$$R = R_{\text{transparent}} e^{+H_0 t} \quad (13)$$

This result shows a radius increasing exponentially with time, and will also result in a straight line on semi-log paper. An $H_0 < 72 \text{ km/s/Mpc}$ is needed to keep $v < c$ at 4.2 Gpc.

Starting with zero velocity, a similar result is obtained by using finite difference techniques to calculate the expansion of the universe from the Hubble acceleration, $a = H^2 R$, obtained by differentiating Hubble's Law. The relativistic expression for adding velocities must be used in this latter analysis. Note that Hubble's law is being applied in a forward direction from the initial fireball, rather than as the distance from our Milky Way galaxy. An initial relativistic velocity, from the time the universe becomes transparent, is needed to obtain a universe age of $\sim 14 \text{ Gyr}$, and this is the issue with using equation (12) – it assumes zero initial speed from the time when the universe became transparent.

Expansion using a Friedmann Equation

Equation (11) can actually be derived from equation (2) by recognizing that the last term of equation (2) will dominate at large R as a Euclidean ($k=0$) universe expands. The first term of equation (2) dominates at early times, first as a radiation pressure causing a rapid expansion, then as an attractive term due to gravity as the matter density dominates over the radiation density. However, the density will fall off as volume expands (as $1/R^3$), so the first term falls off as $1/R$ when matter density dominates. The last term, the cosmological term, increases as R^2 as the universe expands, so will eventually dominate over all other terms. This becomes a de Sitter universe (1917) with the density parameter $\Omega_m = 0$, and the dimensionless lambda parameter $\lambda = 1$, so the deceleration parameter $q_0 = \Omega_m/2 - \lambda = -1$ (indicating an acceleration instead of deceleration), and therefore [see Rowan-Robinson (2004)]:

$$\Lambda = H_0^2 (1 - 2q_0) = 3H_0^2 \quad (14)$$

Substituting these assumptions into equation (2) then matches equation (11) and we will again calculate an age of the universe near 100 billion years. However, it takes a very long time for the cosmological term to dominate, resulting in an overestimate of age, so all terms of equation (2) are needed.

Rowan-Robinson (2004) and Berry (1976) used all terms to calculate the evolution of the universe, after “recombination” or when the universe became transparent, using equation (2). Many different cosmological models and solutions to the age of the universe are discussed; the latest analyses estimate an age of ~ 13.8 billion years, corresponding to an expansion at light speed. It should be noted that this approach also does not agree with Hubble’s Law or redshift data of galaxies.

Berry (1976) has some nice closed form solutions for the age and radius R of the universe for $k=0$. For $\Lambda > 0$, the age is shown in equation (15); however, there is a typographical error in R vs t for $\Lambda \neq 0$, corrected below in equation (16) for $\Lambda > 0$:

$$t = \frac{2}{\sqrt{3\Lambda}} \operatorname{arc} \sinh \left[\left(R/R_o \right)^{3/2} \sqrt{\frac{\Lambda}{8\pi G \rho_o}} \right] \quad (15)$$

$$R = R_o \left(\frac{8\pi G \rho_o}{\Lambda} \right)^{1/3} \sinh \left[\left(0.5 t \sqrt{3\Lambda} \right)^{2/3} \right] \quad (16)$$

where R_o , t_o , and ρ_o refer to current conditions, which are currently believed to be 4.2 Gpc, 13.8 Gyrs, and $\sim 3 \times 10^{-30}$ g/cc including dark matter ($\Omega_m \sim 0.3$), converted of course to consistent units.

Numerical Methods

Setting $R = R_o$ in equation (15) gives the age of the observable universe, dependent only on

density ρ_0 and Λ , which in turn is dependent on Ω_m and Hubble's constant for $k = 0$. Solving equation (2) numerically then predicts the universe's expansion R vs time t . Figure 1 shows the expansion vs time for $k=0$, $H_0 = 72$, $\Omega_m = 0.3$, $\lambda = 0.7$, and $q_0 = -0.55$; however, if the current radius $R_0 = 14.4$ Gpc, as some believe, the actual age would be 32 billion years. Changing H_0 to 68 with $\Omega_m = 0.3$ changes the age to 13.8 Gyrs for $R = 4.2$ Gpc..

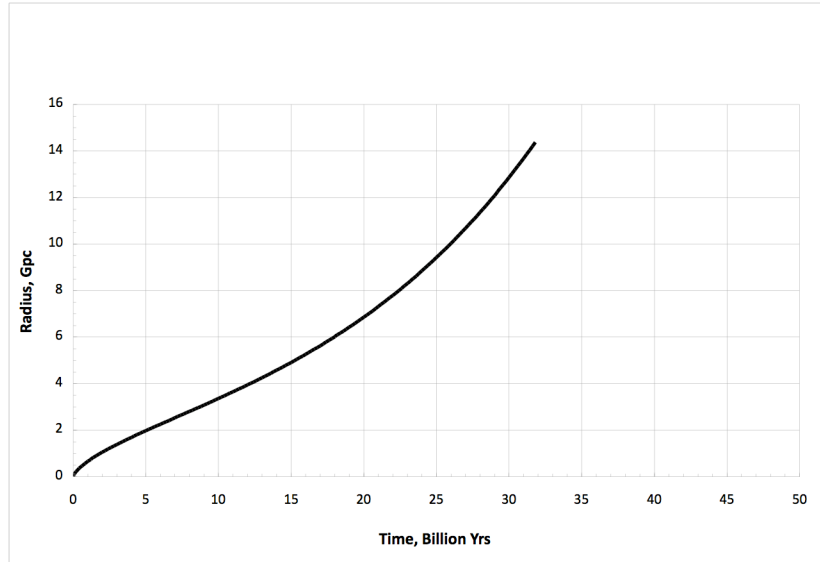


Figure 1. The Universe Expansion vs Time for $H_0 = 72$ and $\Omega_m = 0.3$

Backing out H from Figure 1, starting from $R_0 = 4.2$ Gpc and going backwards, with $D = 4.2 - R$, gives unreasonable values for Hubble's constant as shown in Figure 2. This goes out to verified distances, but does not take into account different definitions of distance given in general relativity, such as proper vs co-moving distances.

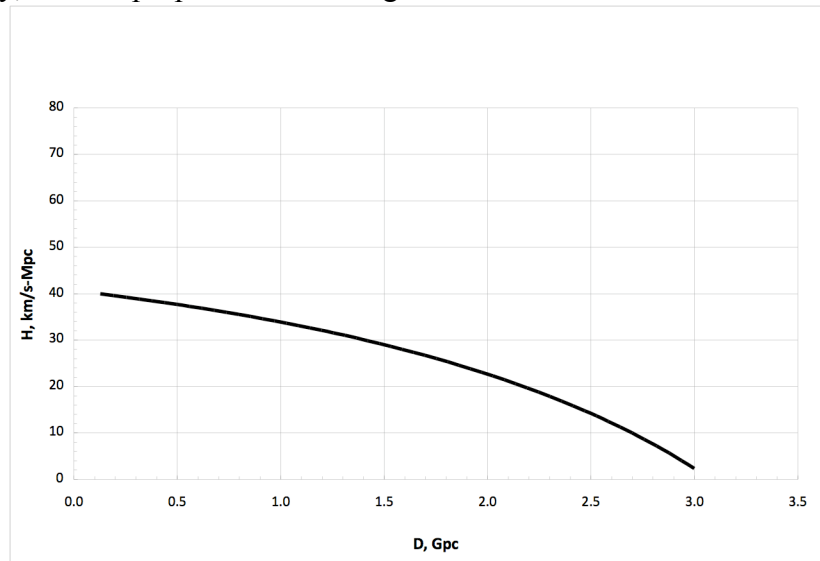


Figure 2. Retrieved H Does Not Match an Input $H_0 = 72$ for $\Omega_m = 0.3$ and $R_0 = 4.2$ Gpc

A reasonable match to H_0 can be obtained by reducing the percent of observable and dark matter to lower values. For example, Figure 3 shows the universe's expansion changing Ω_m to 0.011, which implies $\lambda = 0.99$ and $q_0 = -0.98$. A close match to the input H_0 is shown in Figure 4. The observable universe age is now 27 billion years and it takes 44 billion years to expand to 14.4 Gpc. An even closer match to the input H_0 can be realized for $R_0 = 14.4$ Gpc. Additionally, the expansion speed stays below light speed after the first 0.1 billion years since the Big Bang.

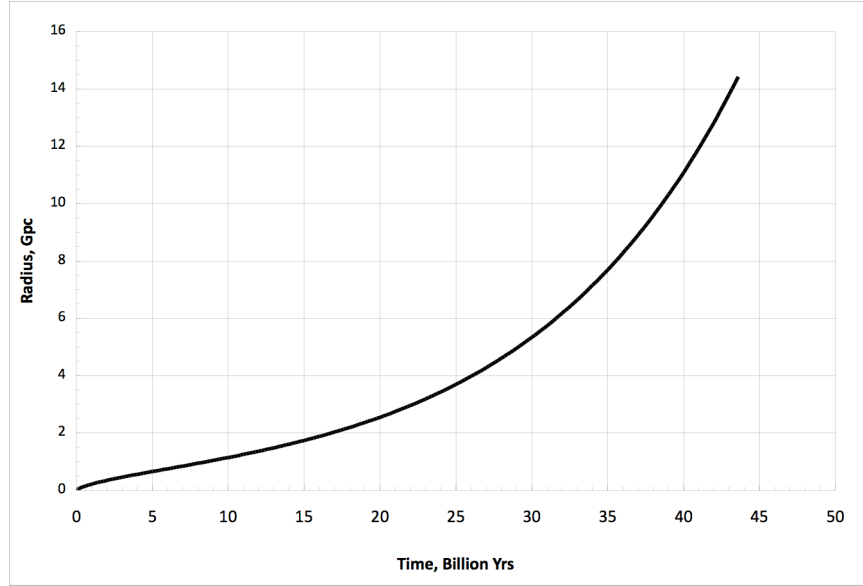


Figure 3. The Universe Expansion vs Time for $H_0 = 72$ and $\Omega_m = 0.011$

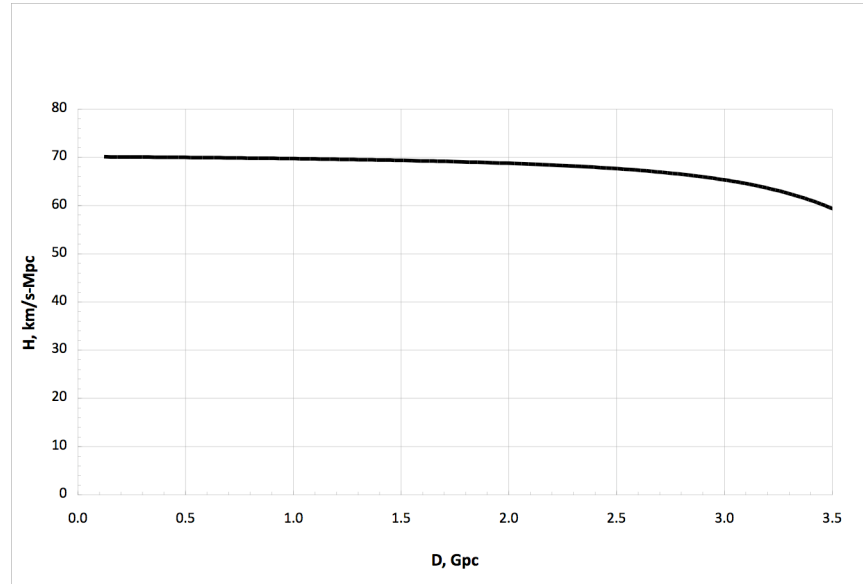


Figure 4. A Retrieved H is Reasonable for an Input $H_0 = 72$, $\Omega_m = 0.011$ and $R_0 = 4.2$ Gpc

For the currently estimated observed mass fraction of $\sim 4\%$, excluding dark matter, a larger universe radius is required to back out a reasonable H. If the current radius is really 14.4 Gpc,

then the age is ~ 37 billion yrs as shown in Figure 5, and the Hubble Space Telescope (HST) has only looked back in time a fraction to the beginning of the Big Bang. In this case, for $\Omega_m = 0.044$, we are able to retrieve very reasonable $H \sim 70$ values versus confirmed distances (about 3000 Mpc from us), $D = 14.4 - R$, as shown in Figure 6. The expansion drops below light speed after 0.5 Gyrs from the Big Bang. Dark matter is not needed although it would not resolve the missing mass problem with galactic rotation curves.

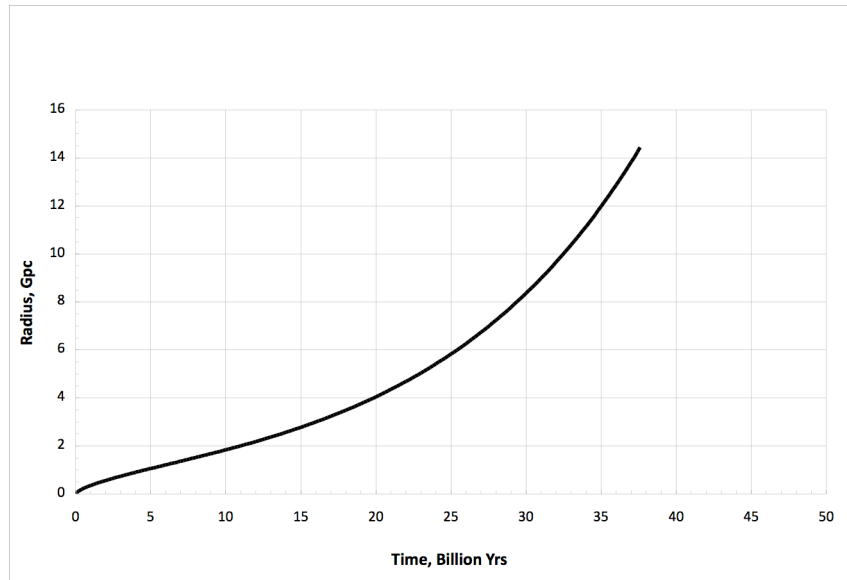


Figure 5. The Universe Expansion vs Time for $H_0 = 72$ and $\Omega_m = 0.044$

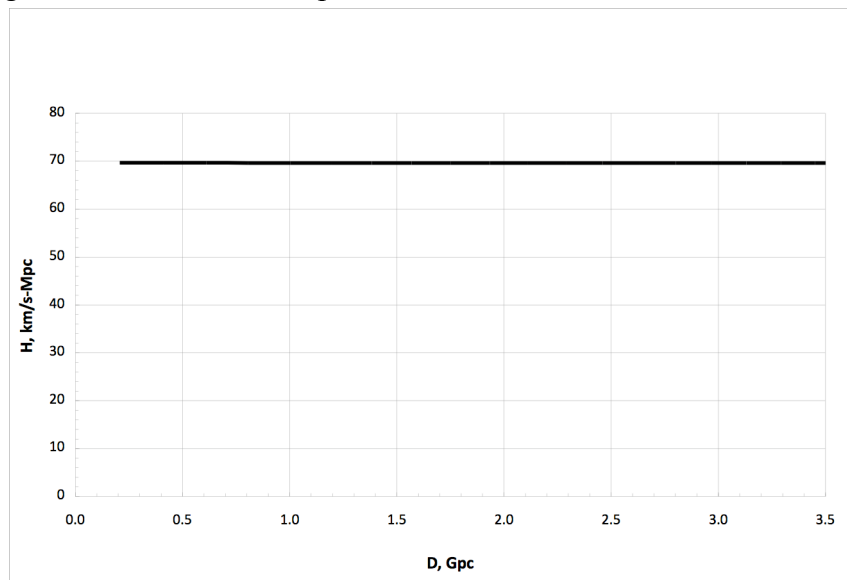


Figure 6. A Retrieved H is Good for an Input $H_0 = 72$, $\Omega_m = 0.044$, and $R_0 = 14.4$ Gpc

Distances in Special Relativity and General Relativity

At galaxy distances out to where the HST can spatially resolve galaxy features, the special relativity distances obtained using Hubble's law and general relativity distances computed using

H_0 , Ω_m , and λ for a Euclidean ($k=0$) universe are very similar. Distances obtained by any of these measures are within a couple percent of each other out to the Coma cluster of galaxies, as shown in Figure 7 as a function of redshift z . By $z=0.1$, general relativity distances are within 7% of the special relativity distances, excepting proper distance which is within 12% [co-moving distance multiplied by the scale factor, $1/(1+z)$]. Hubble (1929) used obsolete redshift and distance data out to NGC4486 (M87) to infer that extragalactic distances were approximately proportional to radial velocity. M87 is now known to be a giant elliptical galaxy at a distance of ~ 53 million light years (~ 16 Mpc), visible as a fuzzy spot in amateur telescopes, in the Virgo cluster of galaxies. M87 has a supermassive black hole, first observed using radio telescopes.

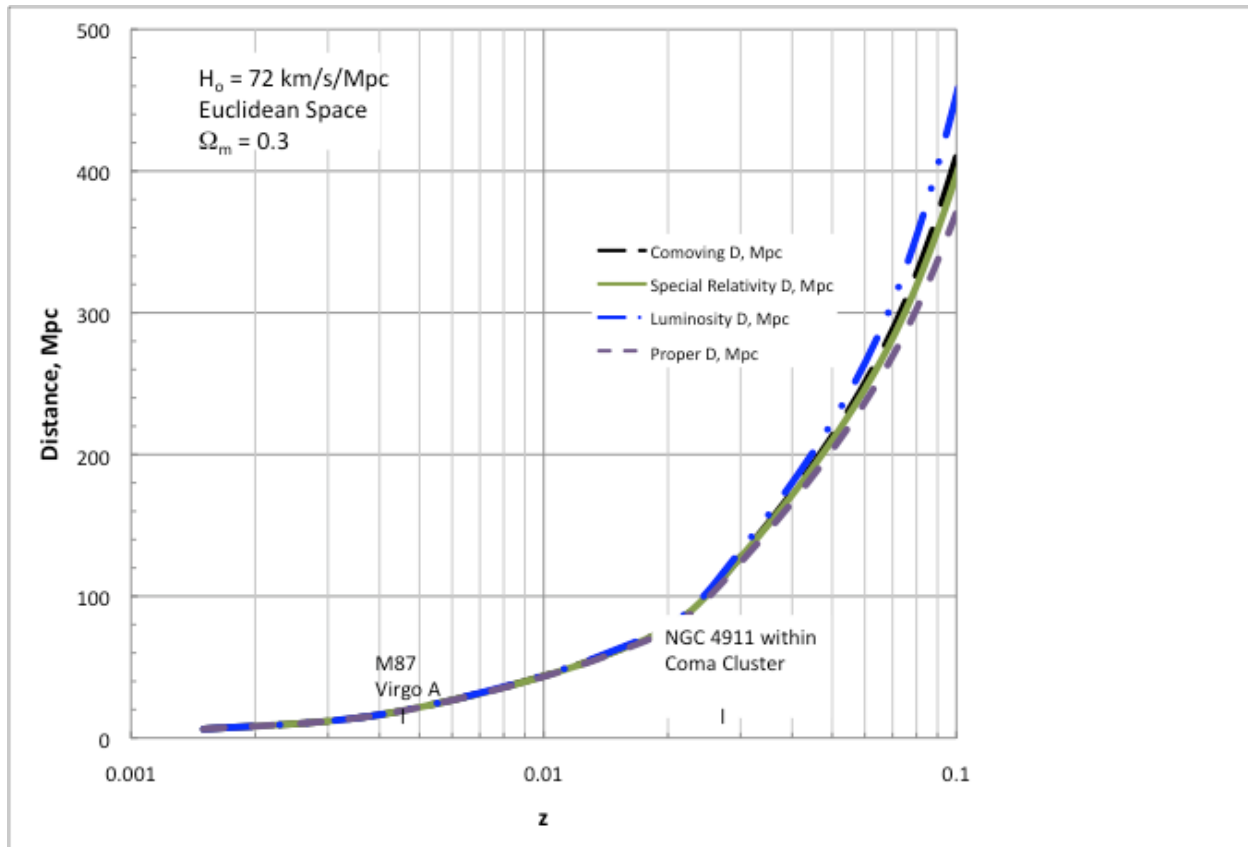


Figure 7. Distance Measures at Lower Redshifts

However, at larger redshifts, distances to galaxies become nebulous! As shown in Figure 8, special relativity (SR) distance levels out at around 4100 Mpc (13.4 billion light years) for an $H_0 = 72$ km/s/Mpc. Assuming a mass fraction of 0.3 and energy content of 0.7, the general relativity (GR) co-moving distance approaches 13.3 Gpc (~ 43 billion light years) for $H_0 = 72$ km/s/Mpc and $\Omega_m = 0.3$ in a Euclidean universe. Slight changes in these values can result in a co-moving distance of 14.4 Gpc (~ 47 billion light years) which some say is the current radius of the universe allowing for continued expansion to the current day. The luminosity distances, which are used to back out distances from high redshift objects, go off scale and appear unrealistic.

Therefore, astronomers typically plot luminosity distance modulus, $m-M$, in terms of redshift only, or $1/(1+z)$, at high redshifts. The apparent magnitude is m and the absolute (intrinsic) magnitude is M . There is some uncertainty in calculating M from optically observable m , due to extinction (absorption and scattering) effects and k factors for spectral distribution shifts vs redshift, as well as errors in Type Ia supernova characteristics. In deducing an “accelerating” universe, Reiss et.al. (1998) analyzed Type Ia supernovae in galaxies from $z=0.16$ to 0.97 . At $z=1$, the luminosity distance is $\sim 2.6X$ higher than the special relativity distance. The HST and the Webb telescopes have now observed galaxies with $z>10$.

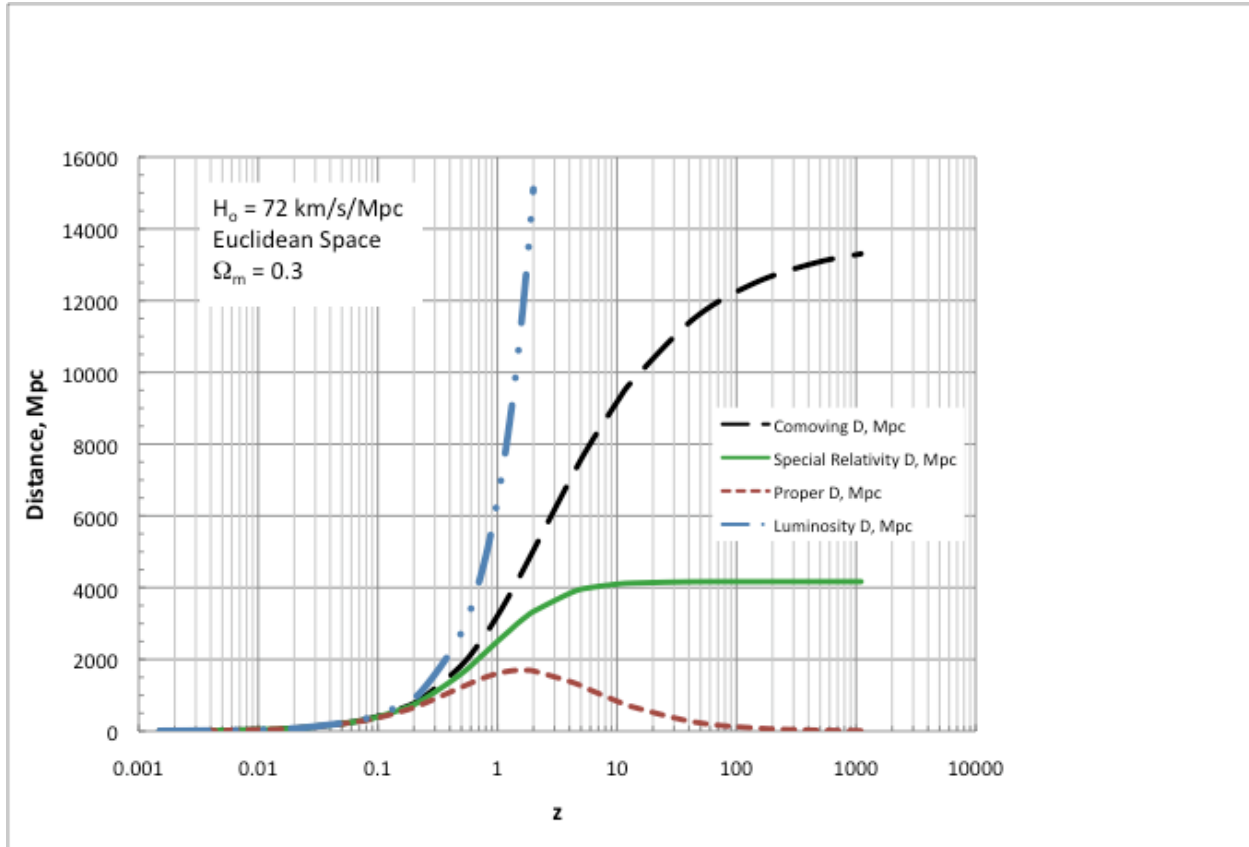


Figure 8. Distance Measures at High Redshifts

Conclusions

The currently estimated fraction of observable and dark matter may be too high. In order to back out reasonable values of Hubble’s “constant” from Friedmann equation calculations, for $R_0 = 4.2$ Gpc, the observable universe age may be greater than 28 billion years but Ω_m could only be approximately one percent. If the universe’s radius is currently 14.4 Gpc, reasonable Hubble “constant” values are backed out with a Ω_m of $\sim 4\%$, and the universe age would become ~ 37 billion years since the Big Bang. If the CMB is not from the Big Bang, then the expansion ratio of ~ 1100 could be much larger, and the size and age of our universe could be much greater.

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Appendix A. Redundant Derivation of the “2nd” Friedmann Equation to Predict Age

If one subtracts the 2nd Friedmann equation from the 1st, the universe’s acceleration rate can be determined, as best shown by Rowan-Robinson (2004):

$$\frac{d^2 R}{dt^2} = \frac{-4\pi G R}{3} \left(\frac{3p}{c^2} + \rho \right) + \frac{\Lambda R}{3} \quad (\text{A-1})$$

Note that the left hand side of the equation is essentially the acceleration of the universe, and that the first term on the right hand side of the equation can be interpreted as a deceleration term that could cause a gravitational collapse, and the last term can lead to deceleration or acceleration of the universe depending on the value of the cosmological constant, as noted by Swihart (1968). As a side note, this equation appears illogical, as the pressure term (which should contribute to an expansion) and the density term (which should contribute to a gravitational collapse) have the same sign. The pressure term can also be interpreted as an equivalent radiation density, where radiation pressure causes an initial rapid expansion and later represents an equivalent radiation density.

The cosmological constant, Λ , is a term added by Einstein (1917) to create a static universe, the predominant thinking at the time, by setting $\Lambda = 4\pi\rho G$ prior to the work of Hubble (1929).

One can substitute an equivalent radiation density for the pressure term, $\rho = \rho_r + \rho_m$, and then rewrite equation (A-1) as:

$$\frac{d^2 R}{dt^2} = \frac{-4\pi G \rho R}{3} + \frac{\Lambda R}{3} \quad (\text{A-2})$$

where ρ now includes radiation, visible matter and dark matter. Assuming conservation of equivalent mass:

$$\rho R^3 = \rho_o R_o^3 \quad (\text{A-3})$$

where the subscript refers to current conditions, we can rewrite equation (A-2) as:

$$\frac{d^2 R}{dt^2} = \frac{-4\pi G \rho_o R_o^3}{3R^2} + \frac{\Lambda R}{3} \quad (\text{A-4})$$

Multiplying both sides by dR/dt and integrating with respect to t :

$$\int \frac{dR}{dt} \frac{d^2 R}{dt^2} dt = \int \frac{dR}{dt} \frac{-4\pi G \rho_o R_o^3}{3R^2} dt + \int \frac{dR}{dt} \frac{\Lambda R}{3} dt \quad (\text{A-5})$$

which becomes using $dR/dt = \dot{R}$ and noting the change in sign:

$$\int \dot{R} \frac{d\dot{R}}{dt} dt = \frac{4\pi G \rho_o R_o^3}{3R} + \int \frac{\Lambda R}{3} dR$$

then integrating and substituting equation (A-3) again:

$$\dot{R}^2 / 2 = \frac{4\pi G \rho R^2}{3} + \frac{\Lambda R^2 / 2}{3} + \text{constant of integration}$$

This assumes Λ is a constant, and it is obvious that equation (2) follows.

Appendix B. Some Comments on Current Beliefs

Cosmologists generally believe that there was no explosion associated with the “Big Bang” and that we are observing the expansion of space itself. I disagree [Spieth (2017)]. Early temperatures and pressures predicted by the same Friedmann equations are consistent with a thermonuclear fusion explosion that would cause a rapid expansion of matter. The reason that we cannot see the origin of this explosion is because it has long ago subsided, and light emissions from it would overtake galaxies and be lost in space. Because almost all galaxies are expanding away from us implies that they were once all together; however, extrapolating this back to a point singularity is unreasonable, as pressure forces would prevent such early densities. General relativity does not have all the physics needed to describe the early expansion of the universe – it does not include electromagnetic nor nuclear fusion effects, and perhaps other effects.

The CMB appears from all directions, not from a single point in space. Tolman (1934) showed that a blackbody will preserve its blackbody characteristics at a lower temperature when redshifted (frequency shift), but did not prove it will radiate from all directions. I believe the CMB was pre-existing prior to the Big Bang, not redshifted light from the Big Bang.

An expanding gas will cool, and re-radiate at discrete wavelengths (not as a blackbody) based on its temperature and composition. Hence, if the CMB is not from the Big Bang, we are not constrained to an expansion ratio of ~ 1100 since the observable universe became transparent, and the age of our observable universe could be much greater.

Hubble's empirical law already implied an accelerating universe, which appears to be accelerating slightly faster than $a = H^2 R$ (by differentiating $v = HR$) based upon work by Riess and Perlmutter. The Einstein and Friedmann equations include pressure, gravity, and a fictitious cosmological constant also referred to as dark energy, but no electromagnetic forces that are known to exist in the universe. Spieth (2017) showed that Hubble's law, and hence an accelerating universe, can be derived assuming a pre-existing charged particle universe that generate electrical forces, into which galaxies with similar charges expand. This would replace the cosmological term. The charge requirements are miniscule and will be difficult to measure. Additionally, it would be naïve to assume there was no matter or energy prior to the Big Bang.

Appendix References

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The following is one of several Excel spreadsheets written by the author to calculate the age of the universe from equation (15). Alternatively, finite difference methods used equation (2).

This calculates the age of the universe using equation (7.2.15) from Berry (1976) for a positive cosmological constant & k=0											
		Age = (2/SQRT(3 x Λ)) x arc sinh [SQRT($\Lambda/(8\pi G\rho)$)]									